Unemployment and Endogenous Growth with New Technologies-Skill Complementarity

E. Moreno-Galbis *

IRES-Université Catholique de Louvain

Abstract

We construct an endogenous growth intertemporal general equilibrium model with two types of jobs and two types of workers. We allow for job competition between high- and low-skilled workers on the low-skilled segment of the labor market and for on-the-job search for high skilled workers. Matching processes are represented by matching functions *à la* Pissarides. Workers search intensities are endogenous. We distinguish between embodied and disembodied technological progress and endogenize them through a *learning by doing* process based on capital accumulation. Social returns to capital are imposed to be constant. Biased technological change is introduced via embodied technical progress and new technologies-skill complementarity relationship. The model reproduces quite well the unemployment rate evolutions, the relative wage stability and the *productivity slowdown puzzle* observed over the last decades. It suggests strong interactions between embodied technological progress, growth, biased technological change, discouragement effects and job competition.

JEL classification: E24, J21, J23, O40

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1 Introduction

Over the last 25 years the economic evolution of most European economies has been characterized by three facts. First, there has been an upturn in unemployment rates, particularly in that of low-skilled workers. This unemployment rise has been accompanied by a rigidity in relative wages. Finally, after the first oil shock, there has been a deceleration in the growth rate of per capita output and in the growth rate of total factor productivity (*productivity slowdown puzzle*[1]). While the two first phenomena are European specific, the last one is observed in most OECD countries. Table 1 shows that in spite of the increase in unemployment rates between 1970 and 2000 in most OECD countries, relative wages have remained quite rigid. Table 2 summarizes data concerning per capita output growth rates using Maddison (1991) data for the period going from 1950-1987 and OECD data for the nineties. Even if both data sources are not strictly comparable (particularly, data on the nineties refers to reunified Germany) it can be suggested that while the productivity slowdown was clearly observed along the seventies and eighties for all countries, during the nineties it seems to have reduced its pace for countries as United States or United Kingdom.

Even if unemployment and growth problems often arise together, the literature has traditionally treated them separately. That is, models of economic growth typically assume full employment and models dealing with employment assume no growth. In this paper we consider both problems together. More particularly we analyze the interactions between unemployment, relative wages and growth observed in European countries over the last 25 years. We build a model based on Moreno-Galbis and Sneessens (2004) on what concerns the labor market side and on Boucekkine, del Rio, and Licandro (2002) regarding the way growth, embodied and disembodied technological progress are introduced. The main contribution of the model here presented is to consider growth issues while keeping, at the same time, a realistic representation of the labor market. We have thus an endogenous growth model in an intertemporal general equilibrium setup where the labor market is characterized by the presence of search and matching frictions.

We differentiate in the labor market between two types of jobs (complex and simple) and two

[1]The deceleration in growth rates is referred as puzzling simply because there is no an unanimously accepted explanation of it.

	UNEMPLOYMENT RATES			RELATIVE WAGES			
	1970-1979	**1980-1989**	**1990-2001**		**1980-1984**	**1985-1989**	**1990-1995**
France	3.9	9.1	10.7	D5/D9	51.7	50.8	50.3
				D1/D5	60.6	61.5	60.9
Germany	1.4	5.2	7.3	D5/D9	60.8	61.0	61.4
				D1/D5	60.0	64.4	67.4
Japan	1.7	2.5	3.4	D5/D9	55.9	54.4	53.9
				D1/D5	58.2	58.5	60.4
United Kingdom	4.3	10.0	6.8	D5/D9	58.2	55.5	53.9
				D1/D5	59.0	56.8	55.9
United States	6.2	7.2	5.5	D5/D9	..	..	48.4
				D1/D5	..	..	48.1

D5/D9: ratio of the upper earnings limit of the fifth decile of workers to the upper limit of the ninth decile. D1/D5: ratio of the upper earnings limit of the first decile of workers to the upper limit of the fifth decile.
Source concerning unemployment rates: For France, Japan, United Kingdom and United States we use LABORSTA database, from the International Labor Office. The data concerning Germany comes from the OECD Economic Outlook.
Source concerning relative wages: OECD Employment Outlook 1996 chapter 3. For Germany we have information only between 1983-1993. For US data concerns only the 1993-1995 period.

Table 1: Average unemployment rates and relative wages in five OECD countries (in percent).

	1950-1973	**1973-1987**	**1990-2000**
France	4.0	1.8	1.4
Germany	4.9	2.1	1.3
Japan	8.0	3.1	1.1
United Kingdom	2.5	1.8	1.9
United States	2.2	1.6	2.2

Source: The two first columns are constructed from Tables 3.3, 3.5, 5.3, 5.4 and 5.19 in Maddison (1991). The last column corresponds to OECD Economic Outlook No.70.

Table 2: Average growth rates of per capita output, 1950-1987, 1990-2000 (percent).

types of workers (high- and low-skilled). We allow for job competition between high- and low-skilled workers on the low-skilled segment of the labor market and for on-the-job search for high-skilled workers. Search intensities are endogenous for all workers and wages are negotiated through a normal Nash bargaining.

The distinction between embodied and disembodied technological progress has been shown to be relevant by a number of recent theoretical and empirical works which have invoked the embodied nature of an increasing fraction of technological progress. Particularly, according to Greenwood, Hercowitz, and Krussel (2002) 60% of US productivity growth comes from embodied technological change. The characteristics of embodied technological progress are estimated to play a key role in explaining some US stylized facts such as the productivity slowdown, the decline in the relative price of investment or the persistent upturn in the equipment to output ratio. Since these stylized facts have also been observed for many European countries, there is a strong belief among economists that embodied technological progress is also behind. We therefore distinguish between both types of technological progress and endogenize them through a *learning by doing* (LBD) process based on capital accumulation[2] that turns out to be the engine of growth.

Social returns to capital are imposed to be constant so that a balanced growth path arises. The productivity slowdown is explained on the basis of a reassignment effect, according to which, after the first oil shock, the upturn in embodied technological progress crowded out the disembodied technical one. This induced, on the one hand, a permanent fall in the relative price of investment[3], thereby increasing the user cost of capital[4], which tended to slow down output growth. On the other hand, the acceleration of embodied technological progress has been associated to an increased relative demand for high-skilled labor, as a result of either technological requirements (see Berman, Bound, and Griliches (1994) for the U.S. and Machin, Ryan, and

[2]The LBD in the final good sector is consistent with Romer (1986) whereas LBD in the production of new investment goods is close to Arrow (1962).

[3]Because the efficiency of the embodied technical progress production process improves at a faster rate than that of consumption goods, the relative price of investment goods falls.

[4]The faster the rate at which the efficiency of the embodied technical progress is improved, the faster the pace at which capital goods become obsolete. This expected loss in the value of capital is included in the usage cost of capital.

Van Reenen (1998) for some European countries) or induced organizational changes (see Caroli and Van Reenen (2001) and Bresnahan, Brynjolfsson, and Hitt (2002)). Technological progress over the last decades has thus been skill-biased. This new technologies-skill complementarity relationship[5] is also represented by the model.

The model is calibrated on the basis of Belgian data and simulated to test its ability to reproduce the evolution of unemployment rates, relative wages and growth rates. Afterwards, we implement some dynamic simulations to analyze the consequences of different policy measures. We conclude that the model provides a good representation of the economy and constitutes the first attempt to reproduce simultaneously these three stylized facts that have characterized most European countries.

The paper is organized as follows. Next section makes a review on the previous literature. In section 3 we present the model. We describe the two forms of endogenous technological progress and determine the presence of a balanced growth path. Finally we describe labor market flows, behaviors of all agents in the model and the wage bargaining process. In section 4 we calibrate the model on Belgian data for 1996. The properties of the model are examined by simulating its responses to various types of shocks. We next set the technological and labor force composition variables to their 1976 values and check the ability of our theoretical setup to reproduce the stylized facts observed in a typical European economy (Belgium). Alternative policy scenarios are also analyzed. Section 5 concludes.

2 Previous literature

The literature has traditionally treated separately unemployment and growth problems. In this sense, concerning the productivity slowdown puzzle, authors like Baily (1981) argue that after the first oil shock, part of the stock of capital, being very energy consuming, was desaccumulated. Denison (1985) points to the negative effects of regulation over economic growth. Blan-

[5] There exists a considerable amount of empirical literature analyzing the importance of the capital-skill and/or new technologies-skill complementarity relationship over the last decades. See Berman, Bound, and Griliches (1994), Fitz Roy and Funke (1995), Machin, Ryan, and Van Reenen (1998), Krusell et al. (2000) or Moreno-Galbis (2002), among others.

chard (1997) defends that with the tertiarisation of the economy the absorption of technological progress has become more difficult and so has growth. More recent and popular arguments (Greenwood and Yorukoglu (1997)) claim that the post-1974 period was characterized by the huge raise of information and communication technologies (embodied technological progress). The expansion of these technologies to the whole economy takes time because of the presence of learning costs (physical and human capital investment), which results in a reduction of the growth rates. Finally, Boucekkine, del Rio, and Licandro (2002) suggest the existence of a technological reassignment after the first oil shock favoring of embodied technological progress and against disembodied technical change. This would have induced a permanent increase in the user cost of capital slowing down productivity growth. None of these models analyzes the impact of growth on unemployment, or the role that unemployment could have had on growth.

On the other hand, there are also many models trying to explain the upturn in European unemployment rates, that do not consider growth issues. Gautier (2002) develops a stylized partial equilibrium model with two types of jobs, two types of workers and wage bargaining. He focuses on the stationary state properties of the model. Dolado, Felgueroso, and Jimeno (2000) use a similar approach (with a simpler albeit more realistic representation of wage determination) to evaluate the job competition effect triggered by the dramatic increase in the proportion of skilled workers that took place in the late eighties in Spain. Similar models are also considered in Albrecht and Vroman (2002) and Dolado, Jansen, and Jimeno (2002). Collard, Fonseca, and Muñoz (2002) provide a first attempt to include this type of quantitative analysis in a dynamic general equilibrium setup. Pierrard and Sneessens (2002) use a similar setup with on-the-job search and endogenous search intensities for high-skilled workers so as to obtain a less mechanical job competition effect. Their model explains a significant part of the unemployment rise observed over the last twenty years. Moreno-Galbis and Sneessens (2004) use an intertemporal general equilibrium setup with two types of jobs, two types of workers, job competition, on-the-job search, endogenous search intensities for all workers and an endogenous capital-skill complementarity relationship. Their model manages to mimic the increase in unemployment rates as well as the relative wage rigidity by simply introducing the observed raise in the proportion of high-skilled workers in the labor force and the decline in the relative prices of new investment goods that has characterized the last decades.

Since the beginning of the nineties there is an emerging literature introducing growth into matching and search models of unemployment or matching and search frictions into growth models. One of the first attempts was made by Pissarides (2000), who analyzed the relationship between growth and unemployment in the presence of embodied and disembodied exogenous technical change. Bean and Pissarides (1993) develop a simple OLG endogenous growth model with matching frictions to show how high unemployment rates can have adverse effects on growth through the reduction in the pool of savings available for investment. Aghion and Howitt (1994) argue that growth occurs through creative destruction of low productivity jobs by high productivity jobs (*reallocation effect*). Merz (1995) manages to capture many of the stylized facts observed in the U.S. labor market using a Real Business Cycle (RBC) model with search frictions and exogenous labor augmenting technological progress. Postel-Vinay (2002) develops a Schumpeterian model to analyze the short- and long-run responses of unemployment to exogenous changes in the rate of technological progress. Postel-Vinay (1998) goes a step further and instead of making only a traditional comparative static analysis, he studies the transitional dynamics of the well known Pissarides (2000) search model in an endogenous growth framework *à la* Romer (1986). More precisely he implements some simulations using a linearized version of the model that turns out to be a good proxy of the "true" dynamics of the model (this talks well about RBC models since they also use linearized models to simulate the true dynamics of a nonlinear model). The growth-unemployment relationship is considered in an efficiency-wage model framework by Brecher, Chen, and Choudhri (2002) and in a generalized augmented Solow type model by Bräuninger and Pannenberg (2002).

3 The Model

The structure of the model is based on Moreno-Galbis and Sneessens (2004). There are two broad categories of agents, households and firms. We distinguish two types of households; each type is defined by the skill level (high or low) of their members. All members of a household supply inelastically one unit of labour; they may be employed or unemployed[6].

[6]The representative household formulation amounts to assuming that workers of a given group are perfectly insured against their own individual unemployment risk. This simplification is common in the literature and

We distinguish three types of firms:

- Intermediate firms producing high-tech intermediate goods. The production of these goods involves complex tasks that can only be carried out by high-skilled workers.

- Intermediate firms producing low-tech intermediate goods. The production of low-tech intermediate goods is made of simpler tasks that can be carried out by both high- or low-skilled workers.

- A representative final firm, which combines capital and the two intermediate goods to produce an homogeneous final good that can be used for consumption or capital accumulation.

There is thus a double heterogeneity as in Gautier (2002), heterogeneity of jobs (complex vs simple) and heterogeneity of workers (high- and low-skilled).

There are three types of markets: labor, goods and capital. On the labor side, we distinguish between the complex and the simple job markets. For each type of job, we assume an exogenous job destruction rate and represent the matching process by a standard matching function. High-skilled unemployed workers may look for both types of jobs. Furthermore, the set of parameter values adopted in the model guarantees the absence of corner solution, *i.e.* : there is always a number of high-skilled workers in simple jobs. High-skilled workers hired on a simple job may continue searching for a complex job (on-the-job search). Because they know that their application will always be turned down, low-skilled job seekers never apply for complex jobs. They may search more or less intensively for a simple job, depending on its attractiveness compared to home production. All goods markets (the two intermediate goods and the final good markets) are assumed to be perfectly competitive. The price of the final good is normalized to one. On the capital market, the supply is determined by the stock of capital previously accumulated by the household. The interest rate adjusts to make the quantity demanded by the representative final firm equal to this predetermined capital stock.

Social returns to capital are imposed to be constant, which is necessary for a balanced growth path to arise. In order to compute the steady state we determine first the growth rate of

reflects the current state of the art. Taking into account the consequences of imperfect insurance markets and workers's *ex post* heterogeneity would make the model totally untractable.

each variable (see appendix). We then destrend the model redefining it in terms of stationary variables (intensive variables). Finally, we calibrate and simulate.

We next successively discuss about technological progress and the balanced growth path. We then define the optimizing behavior of each agent as well as the wage bargaining problem.

3.1 Technological Progress and Balanced Growth Path

Embodied *vs* Disembodied Technological Progress

The production process of the final firm is represented through a constant returns-to-scale Cobb-Douglas function with three inputs:

$$\widetilde{F}\left(\widetilde{K_t}, Q^c_t, Q^s_t\right) = \widetilde{z}_t \left[\widetilde{K_t}\right]^{1-\mu} \left[\left(Q^c_t\right)^{\theta^c_t} \left(Q^s_t\right)^{\theta^s_t}\right]^{\mu} , \qquad \theta^c_t + \theta^s_t = 1 , \tag{1}$$

where Q^c_t and Q^s_t represent, respectively, the intermediate complex and simple goods, $\widetilde{K_t}$ the stock of capital and $\widetilde{z}_t$ disembodied neutral technological progress. Because good markets are assumed to be perfectly competitive and marginal productivity of high-skilled workers is normalized to 1 (one worker produces one unit) and to ν for low-skilled workers (see section 3.3, the intermediate firms' part), the following equalities are satisfied:

$$Q^c_t = N^c_t \qquad \text{and} \qquad Q^s_t = N^{sh}_t + \nu N^{sl}_t , \tag{2}$$

where N^c_t, N^{sh}_t and N^{sl}_t represent, respectively, the employment in complex jobs, the number of simple jobs occupied by high-skilled workers and the number of simple jobs occupied by low-skilled workers.

Embodied technological progress is introduced by allowing new investment goods to be more productive than older ones. Following Boucekkine, del Rio, and Licandro (2002), we write the law of motion of capital as follows[7]:

$$\widetilde{K_{t+1}} - \widetilde{K_t} = \widetilde{e_t}\, \widetilde{I_t} - \delta\, \widetilde{K_t} , \qquad 0 < \delta < 1 , \tag{3}$$

[7]Boucekkine, del Rio, and Licandro (2002) show that this simple representation can be obtained from an explicit vintage model.

where $\widetilde{I}_t$ represents investment expenditures and δ is the exogenous depreciation rate of capital. The variable $\widetilde{e}_t$ (embodied technological progress) is an index measuring the marginal contribution of new investment goods to the aggregate capital stock (so that $1/\widetilde{e}_t$ can be interpreted as the relative price of new capital goods).

Both types of technological progress are determined by a *learning by doing* (LBD) process based on capital accumulation. More formally:

$$\widetilde{e}_t = e_0 \, \widetilde{k}_t^{\,\lambda} \qquad \text{and} \qquad \widetilde{z}_t = z_0 \, \widetilde{k}_t^{\,\gamma} \tag{4}$$

where e_0, z_0, λ and γ are four strictly positive real numbers and $\widetilde{k}_t$ is the capital stock per employed worker. The intuition behind these LBD processes is the following: the more capital is abundant in the economy, the easier it is to innovate, and the more the economy innovates, the more capital will accumulate.

The economic literature distinguishes at least between three types of endogenous growth models using technological innovation as the engine of growth. On the one hand, we have the models in which technological progress shows up as an expansion of the number of varieties of producer and consumer products (see Romer (1987) and Romer (1990)). On the other hand, we have the "creative destruction" models, in which growth arises from improvements in the quality of products (see Aghion and Howitt (1992)). Finally, the LBD models introduced in Romer (1986) assume that a firm that increases its physical capital learns simultaneously how to produce more efficiently (positive effect of experience on productivity). The three types of theoretical setups have as a common point the absence of decreasing returns to scale. They uniquely differ in the way they model technological innovation. In this paper we choose the LBD as an engine of growth because of its simplicity. Moreover, we have an accelerated obsolescence process similar to Aghion and Howitt (1992) and social returns to capital are constant.

Finally, notice that the effects of capital accumulation on technical progress are not internalized by the firms (technological progress is the unintended by-product of capital accumulation). This condition is consistent with the existence of a competitive equilibrium.

Skill-Bias

The empirical evidence suggests that the use of new technologies is associated to an increased relative demand for skilled labor, either because of technological requirements either because induced organizational changes. The variation in the demand for skilled labor should best be seen as resulting from a combination of embodied technological progress and new technologies-skill complementarity. The empirical relevance of these two aspects has been emphasized for instance by Greenwood, Hercowitz, and Krussel (2002) (for embodied technological progress) and by Machin, Ryan, and Van Reenen (1998), Krusell et al. (2000) or Caroli and Van Reenen (2001) (for capital-skill complementarity).

To reproduce this empirical observation, we assume the relative productivity of complex intermediate goods to be an increasing function of two indicators measuring the importance of embodied technological progress. The first indicator is the share of new investment goods affected by embodied technological progress over total capital stock and the second, the efficiency of the embodied technical progress learning process. More formally we assume:

$$\frac{\theta_t^c}{\theta_t^s} = a_0 \left(\frac{\widetilde{e}_t \, \widetilde{I}_t}{\widetilde{K_t}} \right)^{\frac{a_1}{\lambda}} , \qquad a_0, \; a_1 \geq 0 \; . \tag{5}$$

The mechanism captured in equation (5) is the following: increases in the importance of embodied technical progress either through the upturn in the fraction of capital goods incorporating technological progress or through the improvement in the efficiency of the technological progress learning process, increase the relative marginal productivity (in value added terms) of complex intermediate goods and, thus, by equations (2), the relative productivity of complex jobs, which stimulates the demand for high-skilled workers. From equations (1) and (4) we can rewrite equation (5) as:

$$\frac{\theta_t^c}{\theta_t^s} = a_0 \left(z_0 e_0 \, \frac{\widetilde{I}_t}{\widetilde{Y}_t} \right)^{\frac{a_1}{\lambda}} , \qquad a_0, \; a_1 \geq 0 \; . \tag{6}$$

This specification, while being somewhat *ad hoc*, permits, on the one hand, to make a link between the growth side and the labor market side of the model. On the other hand, it provides a simple mean to represent the empirically tested new technologies-skill complementary relationship[8]. Technology induced changes in the values of θ_t^c and θ_t^s modify the relative marginal

[8]One might think that a capital-skill complementarity hypothesis making $\frac{\theta_t^c}{\theta_t^s}$ an increasing function of capital

productivity (in value added terms) of complex and simple jobs and thus the relative productivity of high-skilled workers.

The Balanced Growth Path

Provided social returns to capital are constant, i.e: $\mu = \lambda + \gamma$ or $1 = 1-\mu+\lambda+\gamma$, a balanced growth path can be shown to exist (see chapters 4 and 5 of Barro and Sala-i Martin (1995) for a reminder on the endogenous growth models characterized by the absence of diminishing returns to capital (AK models, more precisely)). Constant social returns to capital imply the constancy of the total efficiency of the LBD process. Therefore, any variation in μ, λ or γ leads to a technological reassignment (the relative importance of each type of technological progress is modified).

Boucekkine, del Rio, and Licandro (2002) remark that if *the initial capital stock (K_0), the consumption-output level ($\widetilde{C_t}/\widetilde{Y_t}$) and the constant growth rate (g) are positive, and if the utility is bounded, then, there exists a unique solution path in which consumption and capital grow at rates g and $g_{\widetilde{k}}$, respectively.* Because of the complexity of our model, rather than establishing the analytical conditions ensuring the positivity of K_0, $\widetilde{C_t}/\widetilde{Y_t}$ and g, we will simply calibrate the parameters so that these conditions are satisfied. Furthermore, since we work with a logarithmic utility function, the positivity of consumption also guarantees the bounded utility. Therefore all required conditions for an AK model are satisfied and a unique solution path arises. We compute it in the appendix.

The main characteristics of the balanced growth path, can be easily deduced in an intuitive way without requiring exhaustive computations. In the equilibrium path, output ($\widetilde{Y_t}$), consumption ($\widetilde{C_t}$) and investment ($\widetilde{I_t}$) must grow at a same rate. From the account identity,

$$\widetilde{Y_t} - \widetilde{v_t} = \widetilde{C_t} + \widetilde{I_t} \tag{7}$$

we deduce that, if $\widetilde{Y_t}$, $\widetilde{C_t}$ and $\widetilde{I_t}$ grow at a constant rate g, the vacancy costs ($\widetilde{v_t}$) must also grow at rate g for a balanced growth path to arise.

accumulation, would be more appropriate. However, in an endogenous growth context this hypothesis leads to an explosive solution, since in the equilibrium balanced growth path, capital keeps continuously increasing and therefore, the relative share of complex jobs in the production function will tend towards infinity.

Vacancy costs are traditionally indexed either to the marginal productivity of labor or to the wages (see Pissarides (2000)). In this work we index them to wages. Because output is determined by a Cobb-Douglas production function, the marginal productivity of labor also grows at g, which leads wages to raise at rate g too. Moreover, because unemployment benefits are defined as a fraction bu (replacement ratio) of the average wage, unemployment benefits increase also at the constant rate g. The indexation of the vacancy costs together with the fact that wages and unemployment benefits grow at rate g at the equilibrium path imply that the employment and unemployment levels, remain constant at the equilibrium. Using equation (1) we compute the equilibrium growth rates of all variables involved in the production of the final good:

- $g_{\widetilde{Y}} = g_{\widetilde{F_{N_t^c}}} = g_{\widetilde{F_{Nsh}}} = g_{\widetilde{F_{Nsl}}} = g$, and
- $g_{\widetilde{K}} = \frac{g}{1-\lambda}$, $g_{\widetilde{e}} = \lambda \, \frac{g}{1-\lambda}$ and $g_{\widetilde{z}} = \gamma \, \frac{g}{1-\lambda}$.

Finally, remark that the marginal utility of the household is defined as $1/\widetilde{c}_t$ (the utility function is logarithmic). This means that marginal utility decreases at the same rate as the wages increase. As a result, it can be expected that the additional instantaneous utility obtained by a household from having one additional member employed, remains constant at the equilibrium path.

We describe now the intensive variables[9] that will be used all through the analysis. We signal with "$\widetilde{\ }$" trended variables while we keep the normal notation when referring to intensive variables. Relative consumption of high- and low-skilled households is respectively defined as:

$$C_t^h = \frac{\widetilde{C_t^h}}{\widetilde{Y}_t} \quad \text{and} \quad C_t^l = \frac{\widetilde{C_t^l}}{\widetilde{Y}_t} \ .$$

We denote the relative vacancy costs associated to the opening of complex and simple vacancies as:

$$\begin{aligned} b_t^c &= \frac{\widetilde{b_t^c}}{\widetilde{Y}_t} \ , \ b_t^s = \frac{\widetilde{b_t^s}}{\widetilde{Y}_t} \quad \text{for the unitary costs and} \\ v_t^c &= \frac{\widetilde{v_t^c}}{\widetilde{Y}_t} \ , \ v_t^s = \frac{\widetilde{v_t^s}}{\widetilde{Y}_t} \quad \text{for the aggregate costs .} \end{aligned}$$

[9]The concept of intensive variable is quite standard in the economic growth literature and it refers to the ratio between two variables raising at the same rate in the equilibrium path, *i.e.* the ratio of two trended variables. The intensive variables are, thus, stationary at the equilibrium.

The wages paid in complex jobs, in simple jobs occupied by high-skilled workers, in simple jobs occupied by low-skilled workers and the average wage paid in a simple job are defined as:

$$w_t^c = \frac{\widetilde{w_t^c}}{\widetilde{Y}_t} \ , \quad w_t^{sh} = \frac{\widetilde{w_t^{sh}}}{\widetilde{Y}_t} \quad w_t^{sl} = \frac{\widetilde{w_t^{sl}}}{\widetilde{Y}_t} \quad \text{and} \quad \overline{w_t^s} = \frac{N_t^{sh}}{N_t^{sh} + N_t^{sl}} \, w_t^{sh} + \frac{N_t^{sl}}{N_t^{sh} + N_t^{sl}} \, w_t^{sl} \ ,$$

while the unemployment benefit is given by:

$$w_t^u = bu \left[\frac{N_t^c}{N_t} \, w_t^c \ + \ \frac{N_t^{sh}}{N_t} \, w_t^{sh} \ + \ \frac{N_t^{sl}}{N_t} \, w_t^{sl} \right] \ .$$

bu representing the replacement ratio and N_t the total employment ($N_t = N_t^c + N_t^{sh} + N_t^{sl}$). Notice that the unemployment benefit is assumed to be at the same level for all workers.

The marginal increase in the surplus obtained by a firm when filling a complex vacancy, a simple vacancy with a high-skilled worker or a simple vacancy with a low-skilled worker is given by:

$$W_{N_t^c}^F = \frac{\widetilde{W_{N_t^c}^F}}{\widetilde{Y}_t} \ , \quad W_{N_t^{sh}}^F = \frac{\widetilde{W_{N_t^{sh}}^F}}{\widetilde{Y}_t} \quad \text{and} \quad W_{N_t^{sl}}^F = \frac{\widetilde{W_{N_t^{sl}}^F}}{\widetilde{Y}_t} \ .$$

Finally, the price at which intermediate complex and simple goods are sold and the usage cost of capital are respectively denoted as:

$$c_t^c = \frac{\widetilde{c_t^c}}{\widetilde{Y}_t} \ , \quad c_t^s = \frac{\widetilde{c_t^s}}{\widetilde{Y}_t} \quad \text{and} \quad c_t^K = \frac{\widetilde{c_t^K}}{\widetilde{Y}_t} \ .$$

3.2 Labor Market Flows

Let N_t^c and N_t^s represent total employment in complex and simple jobs respectively. Simple jobs can be occupied by high- (N_t^{sh}) or low-skilled (N_t^{sl}) workers, so that $N_t^s = N_t^{sh} + N_t^{sl}$, where, as already stated, the set of adopted parameter values guarantees $N_t^{sh} > 0$. Let U_t^h and U_t^l denote the number of high- and low-skilled unemployed job seekers respectively. Normalizing the total labor force to one and denoting α the exogenous[10] proportion of high-skilled workers in the total labor force yields the following accounting identities:

$$N_t^c + N_t^{sh} + U_t^h = \alpha, \qquad \text{and} \qquad N_t^{sl} + U_t^l = 1 - \alpha, \tag{8}$$

[10] An endogenous α would require the model to consider human capital formation and education issues, which is above the scope of the present study.

The number of complex and simple job matches (denoted by M_t^c and M_t^s respectively) is a function of the number of corresponding job vacancies (V_t^c and V_t^s) and effective job seekers (number of job seekers weighted by their search efficiencies), that is:

$$M_t^c = M^c\left(V_t^c\ ,\ sc_t\ U_t^h + so_t\ N_t^{sh}\right) \qquad \text{and} \qquad M_t^s = M^s\left(V_t^s\ ,\ sh_t\ U_t^h + sl_t\ U_t^l\right). \quad (9)$$

Both functions are assumed to be linear homogeneous.

We assume that every day (or every period), both, the high- and the low-skilled unemployed workers, spend a certain number of hours having what we call an *active life*. In words, a constant fraction of their day is spent on searching for a job or doing other productive activities (in our case, domestic production), while the rest of the time is devoted to non-productive activities (such as sleeping, eating, etc.). We assume that the amount of hours spent in *active life* is the same for high- and low-skilled workers, and we normalize it to one.

Given the conditions prevailing on the labor market (wages, probabilities to find jobs, etc.), a high-skilled unemployed allocates its *active* time between searching for a complex job ($0 \leq eh_t \leq 1$) and searching for a simple job ($0 \leq 1 - eh_t \leq 1$). Since the worker knows that he has a positive probability of being hired in any of the two types of jobs, he spends all his *active* time searching in both labor market segments. Moreover, a high-skilled working on a simple job spends a fraction ($0 \leq eo_t \leq 1$) of its leisure (normalized to 1) searching for a complex job. Besides, a low-skilled unemployed splits its *active* time between searching for a job in the simple segment ($0 \leq el_t \leq 1$) and staying at home doing domestic activities ($0 \leq 1 - el_t \leq 1$). Because they know that their application will always be turned down, low-skilled job seekers never apply for complex jobs, so when they are not looking for a job in the simple segment, they simply stay at home doing domestic activities. We have therefore an asymmetry in the behavior of high- and low-skilled workers that results from the asymmetry between complex jobs, which can only be occupied by high-skilled workers, and simple jobs, which can be occupied by both types of workers. Search efficiencies, so_t, sc_t, sh_t and sl_t are concave and increasing functions of the search efforts, eo_t, eh_t, $1 - eh_t$ and el_t, respectively.

We denote labor market tensions by ϑ_t^c and ϑ_t^s respectively, where:

$$\vartheta_t^c \equiv \frac{V_t^c}{sc_t\ U_t^h + so_t\ N_t^{sh}} \qquad \text{and} \qquad \vartheta_t^s \equiv \frac{V_t^s}{sh_t\ U_t^h + sl_t\ U_t^l}. \quad (10)$$

With linear homogeneous matching functions, the probabilities of finding a complex or a simple job per unit of search intensity can be respectively written as follows:

$$p_t^c = \frac{M_t^c}{sc_t\, U_t^h + so_t\, N_t^{sh}} = p^c\left(\vartheta_t^c\right) \qquad \text{and} \qquad p_t^s = \frac{M_t^s}{sh_t\, U_t^h + sl_t\, U_t^l} = p^s\left(\vartheta_t^s\right). \tag{11}$$

The probabilities of filling a complex and a simple job vacancy are similarly given by:

$$q_t^c = \frac{M_t^c}{V_t^c} = q^c\left(\frac{1}{\vartheta_t^c}\right) \qquad \text{and} \qquad q_t^s = \frac{M_t^s}{V_t^s} = q^s\left(\frac{1}{\vartheta_t^s}\right). \tag{12}$$

The probability that a simple job is filled is the sum of the probabilities of hiring a high-skilled worker and a low-skilled worker:

$$q_t^{sh} = \frac{sh_t\, U_t^h}{sh_t\, U_t^h + sl_t\, U_t^l}\, q_t^s \qquad \text{and} \qquad q_t^{sl} = \frac{sl_t\, U_t^l}{sh_t\, U_t^h + sl_t\, U_t^l}\, q_t^s. \tag{13}$$

We assume two exogenous job destruction rates χ^c (for the complex jobs) and χ^s (for the simple jobs), implying for each type of job and worker the following employment dynamics (in terms of vacancies and job-seekers' search effort respectively):

$$\begin{aligned} N_{t+1}^c &= \left(1-\chi^c\right) N_t^c + q_t^c\, V_t^c, && \text{(14-a)} \\ &= \left(1-\chi^c\right) N_t^c + p_t^c\, \left[sc_t\, U_t^h + so_t\, N_t^{sh}\right]. && \text{(14-b)} \end{aligned}$$

$$\begin{aligned} N_{t+1}^{sh} &= \left(1-\chi^s - so_t\, p_t^c\right) N_t^{sh} + q_t^{sh}\, V_t^s, && \text{(15-a)} \\ &= \left(1-\chi^s - so_t\, p_t^c\right) N_t^{sh} + p_t^s\, sh_t U_t^h. && \text{(15-b)} \end{aligned}$$

$$\begin{aligned} N_{t+1}^{sl} &= \left(1-\chi^s\right) N_t^{sl} + q_t^{sl}\, V_t^s, && \text{(16-a)} \\ &= \left(1-\chi^s\right) N_t^{sl} + p_t^s\, sl_t U_t^l. && \text{(16-b)} \end{aligned}$$

Figure 1 summarizes these labor market flows and transition probabilities.

Finally, notice that employment flow variables (number of vacancies, (un)employment level, probabilities of finding a job or of filling a vacancy, search intensities, number of matches) remain constant in the equilibrium balanced growth path.

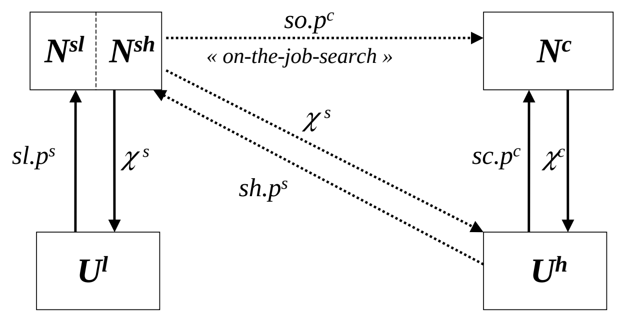

Figure 1: Labor market flows and transition probabilities

3.3 Behaviors

Intermediate Firms

Intermediate firms use only one type of input, high-skilled labor for complex jobs, and high- or low-skilled labor for simple jobs. We further assume that one firm employs only one worker and produces one unit (ν units in case the vacancy is filled by a low-skilled worker) of intermediate good. We thus keep the traditional structure one-job-one-firm which permits to represent the process of wage negotiation through a Nash bargaining sharing rule.

An intermediate firm can open either a complex or a simple vacancy. The unitary cost of opening a vacancy is denoted $\widetilde{b_t^c}$ and $\widetilde{b_t^s}$ for a complex and simple vacancy respectively. These costs are indexed on the corresponding wage rate:

$$\widetilde{b_t^c} = v_0^c \, \widetilde{w_t^c} \qquad \widetilde{b_t^s} = v_0^s \, \widetilde{\widetilde{w_t^s}} \tag{17}$$

where, v_0^c and v_0^s represent strictly positive real constants, $\widetilde{w_t^c}$ the wage paid in a complex job and $\widetilde{\widetilde{w_t^s}}$ the average wage paid in a simple job (since this type of jobs can be occupied either by a high- or a low-skilled worker, and each one earns a different wage, we decided to index simple vacancy costs to the average wage). The aggregate costs of opening complex and simple vacancies are given by :

$$\widetilde{v_t^c} = v_0^c \, V_t^c \, \widetilde{w_t^c} \qquad \widetilde{v_t^s} = v_0^s \, V_t^s \, \widetilde{\widetilde{w_t^s}} \tag{18}$$

where V_t^c and V_t^s stand for the number of complex and simple job vacancies.

The asset value of a complex (resp. simple) unfilled vacancy is denoted as $W_{N_t^c}^V$ (resp. $W_{N_t^s}^V$).

The asset value of a filled complex (resp. simple) vacancy is denoted as $\widetilde{W^F_{N^c_t}}$ (resp. $\widetilde{W^F_{N^{sh}_t}}$ when the simple vacancy is filled by a high-skilled worker or $\widetilde{W^F_{N^{sl}_t}}$ when it is filled by a low-skilled).

$$
\begin{aligned}
W^V_{N^c_t} &= -\widetilde{b^c_t} + \mathrm{E}_t\left[\frac{1}{1+r_{t+1}}\left(q^c_t\,\widetilde{W^F_{N^c_{t+1}}} + (1-q^c_t)\,W^V_{N^c_{t+1}}\right)\right], &(19)\\
\widetilde{W^F_{N^c_t}} &= \widetilde{c^c_t} - \widetilde{w^c_t} + \mathrm{E}_t\left[\frac{1}{1+r_{t+1}}\left((1-\chi^c)\,\widetilde{W^F_{N^c_{t+1}}} + \chi^c\,W^V_{N^c_{t+1}}\right)\right], &(20)\\
W^V_{N^s_t} &= -\widetilde{b^s_t} + \mathrm{E}_t\left[\frac{1}{1+r_{t+1}}\left(q^{sh}_t\,\widetilde{W^F_{N^{sh}_{t+1}}} + q^{sl}_t\,\widetilde{W^F_{N^{sl}_{t+1}}} + (1-q^{sh}_t-q^{sl}_t)\,W^V_{N^s_{t+1}}\right)\right], &(21)\\
\widetilde{W^F_{N^{sh}_t}} &= \widetilde{c^s_t} - \widetilde{w^{sh}_t} + \mathrm{E}_t\left[\frac{1}{1+r_{t+1}}\left((1-\chi^s-p^c_t\,so_t)\,\widetilde{W^F_{N^{sh}_{t+1}}} + (\chi^s+p^c_t\,so_t)\,W^V_{N^s_{t+1}}\right)\right], &(22)\\
\widetilde{W^F_{N^{sl}_t}} &= \nu\,.\,\widetilde{c^s_t} - \widetilde{w^{sl}_t} + \mathrm{E}_t\left[\frac{1}{1+r_{t+1}}\left((1-\chi^s)\,\widetilde{W^F_{N^{sl}_{t+1}}} + \chi^s\,W^V_{N^s_{t+1}}\right)\right]. &(23)
\end{aligned}
$$

where $\widetilde{c^c_t}$ and $\widetilde{c^s_t}$ stand, respectively, for the price of complex and simple intermediate goods; $\widetilde{w^{sh}_t}$ and $\widetilde{w^{sl}_t}$ represent the wages paid in a simple job to a high- and a low-skilled worker, respectively. At the equilibrium, the discount factor $\frac{1}{(1+r_{t+1})}$ is shown to be equal to $\frac{\beta}{(1+g)}$ (see the high-skilled household's part), where β is a psychological discount factor and g the equilibrium growth rate. Variables $W^V_{N^c_t}$ and $W^V_{N^s_t}$ are not signaled as trended since in the equilibrium the free entry condition applies: $W^V_{N^c_t} = W^V_{N^s_t} = 0$. We can now rewrite the intermediate firm equilibrium conditions in terms of intensive variables:

$$
\begin{aligned}
b^c_t &= \mathrm{E}_t\left[\beta\,q^c_t\,W^F_{N^c_t}\right], &(24)\\
W^F_{N^c_t} &= c^c_t - w^c_t + \mathrm{E}_t\left[\beta\,(1-\chi^c)\,W^F_{N^c_t}\right], &(25)\\
b^s_t &= \mathrm{E}_t\left[\beta\left(q^{sh}_t\,W^F_{N^{sh}_t} + q^{sl}_t\,W^F_{N^{sl}_t}\right)\right], &(26)\\
W^F_{N^{sh}_t} &= c^s_t - w^{sh}_t + \mathrm{E}_t\left[\beta\,(1-\chi^s-p^c_t\,so_t)\,W^F_{N^{sh}_t}\right], &(27)\\
W^F_{N^{sl}_t} &= \nu\,.\,c^s_t - w^{sl}_t + \mathrm{E}_t\left[\beta\,(1-\chi^s)\,W^F_{N^{sl}_t}\right]. &(28)
\end{aligned}
$$

Final Firms

The representative final firm uses capital ($\widetilde{K_t}$), complex and simple intermediate goods (Q^c_t and Q^s_t respectively) to produce a final good *via* a linear homogeneous production function $F\big(\widetilde{K_t}, Q^c_t, Q^s_t\big)$. The firm's optimization problem can be represented by:

$$
\max_{\widetilde{K_t},\,Q^c_t,\,Q^s_t}\ \widetilde{F}\big(\widetilde{K_t}, Q^c_t, Q^s_t\big) - \widetilde{c^K_t}\,\widetilde{K_t} - \widetilde{c^c_t}\,Q^c_t - \widetilde{c^s}_t\,Q^s_t\ , \qquad (29)
$$

where $\widetilde{c_t^K}$ stands for the capital usage cost. The first optimality conditions are given by the standard marginal productivity conditions:

$$\widetilde{F_{\widetilde{K_t}}} = \widetilde{c_t^K} \ , \qquad \widetilde{F_{Q_t^c}} = \widetilde{c_t^c} \ , \qquad \widetilde{F_{Q_t^s}} = \widetilde{c_t^s} \ , \tag{30}$$

where $\widetilde{F_{X_t}}$ represents for the first derivative of $\widetilde{F}$ with respect to X_t.

Households

The household decisions bear on consumption and savings and on job search intensities. To avoid untractable *ex post* heterogeneity issues, we assume that workers are perfectly insured against individual unemployment risks by considering two large representative households differentiated by their skill level: a high- and a low-skilled household. Because employment probabilities and expected wage incomes differ among them, their search behaviors, as well as their negotiated wages will be affected. We assume to simplify that the whole capital stock is owned by the high-skilled (high-income) household. This implies assuming that the low-skilled household consumes its current income in every period.

The representative high-skilled household

The members of the high-skilled household can be either unemployed, employed on a complex job paying a wage $\widetilde{w_t^c}$, or employed on a simple job paying a wage $\widetilde{w_t^{sh}}$. The household decides on the consumption level of each of its members, on the fraction of time, eh_t that the unemployed spend searching on the complex segment and on the fraction of the leisure time, eo_t, that employed on low-paid jobs devote to on-the-job-search. The optimization problem of the representative high-skilled household can be written as the following Bellmann equation:

$$\mathcal{W}_t^H = \max_{\widetilde{C_t^h},\, eh_t,\, eo_t} \left\{ \alpha\, \mathcal{U}\left(\frac{\widetilde{C_t^h}}{\alpha}\right) - N_t^{sh}\, \mathcal{D}\left(eo_t\right) + \beta\, \mathrm{E}_t\Big[\mathcal{W}_{t+1}^H\Big] \right\}, \tag{31}$$

subject to constraints (8), (14-b), (15-b) and to the flow budget constraint :

$$\widetilde{w_t^c} N_t^c + \widetilde{w_t^{sh}} N_t^{sh} + \widetilde{w_t^u} U_t^h + \widetilde{c_t^k}\, \widetilde{K_t} + \widetilde{\Pi}_t = \frac{1}{\widetilde{e}_t} \left(\widetilde{K_{t+1}} - (1-\delta)\widetilde{K_t}\right) + \widetilde{C_t^h} + \widetilde{T}_t. \tag{32}$$

$\mathcal{W}_t^H$ is a function of the initial values of the three state variables $\widetilde{K_t}, N_t^c, N_t^{sh}$; $\mathcal{U}(.)$ is an increasing and concave function of per capita consumption ($\widetilde{C_t^h}$ thus measures the total consumption of the high-skilled household); $\mathcal{D}(.)$ is an increasing and convex function of the amount of leisure time devoted to on-the-job search (desutility function); β is a psychological discount factor. The resources of the high-skilled household include wage incomes, an unemployment benefit $\widetilde{w_t^u}$, the rents from capital plus the profits $\widetilde{\Pi_t}$ redistributed by the intermediate goods firms. Investment expenditures are equal to net capital accumulation times the relative price of new capital goods $1/\widetilde{e_t}$ (see equation (3)). $\widetilde{T_t}$ represents a lump-sum tax levied on the high-skilled household to finance government expenditures (such as unemployment benefits, policies consisting in giving subsidies to, for example, low-skilled wages, etc.).

The first-order optimality conditions are given by:

$$\begin{aligned}
\mathcal{U}_{\widetilde{C_t^h}} &= \beta\, \mathrm{E}_t \left[\left(1 + r_{t+1}\right) \mathcal{U}_{\widetilde{C_{t+1}^h}} \right], && (33)\\
0 &= \mathrm{E}_t \left[p_t^c\, sc_{eh_t}\, \beta\, \mathcal{W}^H_{N^c_{t+1}} - p_t^s\, sh_{1-eh_t}\, \beta\, \mathcal{W}^H_{N^{sh}_{t+1}} \right], && (34)\\
\mathcal{D}_{eo_t} &= \beta\, p_t^c\, so_{eo_t}\, \mathrm{E}_t \left[\left(\mathcal{W}^H_{N^c_{t+1}} - \mathcal{W}^H_{N^{sh}_{t+1}} \right) \right], && (35)
\end{aligned}$$

From the envelope theorem, we can obtain the following additional dynamic relationships:

$$\begin{aligned}
\mathcal{W}^H_{N^c_t} &= \mathcal{U}_{\widetilde{C_t^h}} \left(\widetilde{w_t^c} - \widetilde{w_t^u} \right) + \beta \left(1 - \chi^c - p_t^c\, sc_t\right) \mathrm{E}_t \Big[\mathcal{W}^H_{N^c_{t+1}} \Big] && (36)\\
&\quad - \beta\, p_t^s\, sh_t\, \mathrm{E}_t \Big[\mathcal{W}^H_{N^{sh}_{t+1}} \Big]; && (37)\\
\mathcal{W}^H_{N^{sh}_t} &= \mathcal{U}_{\widetilde{C_t^h}} \left(\widetilde{w_t^{sh}} - \widetilde{w_t^u} \right) - \mathcal{D}\left(eo_t\right) + \beta\, p_t^c \left(so_t - sc_t\right) \mathrm{E}_t \Big[\mathcal{W}^H_{N^c_{t+1}} \Big] && (38)\\
&\quad + \beta \left(1 - \chi^s - so_t\, p_t^c - p_t^s\, sh_t\right) \mathrm{E}_t \Big[\mathcal{W}^H_{N^{sh}_{t+1}} \Big]. && (39)
\end{aligned}$$

where $\mathcal{W}^H_{N^c_t} = \frac{\partial \mathcal{W}^H_t}{\partial N^c_t}$ and $\mathcal{W}^H_{N^{sh}_t} = \frac{\partial \mathcal{W}^H_t}{\partial N^{sh}_t}$ represent, respectively, the value of the instantaneous utilities obtained by the high-skilled household from having one additional member employed in a complex or a simple job.

From equation (33) it can easily be obtained the value of the discount factor:

$$1 + r_{t+1} = \frac{1}{\beta} \left(1 + g_{\widetilde{C^h_{t+1}}}\right). \tag{40}$$

Since at the balanced growth path $g_{\widetilde{C^h}} = g$ in the equilibrium we have $1 + r = \frac{1}{\beta}(1 + g)$. Using also equation (33) we determine the endogenous growth rate of the model (Euler equation):

$$g = \frac{\beta}{(1 + g_{\widetilde{e}})} \left[\, z_0\, e_0\, (1 - \mu) \;-\; \frac{(1 - \beta)}{\beta}\, (1 + g_{\widetilde{e}}) \;-\; \delta \;-\; g_{\widetilde{e}} \,\right] \tag{41}$$

where, in the equilibrium path, $g_{\widetilde{e}} = \lambda \, g_{\widetilde{K}}$ and $g_{\widetilde{K}} = \frac{g}{1-\lambda}$ (see appendix). The main difference in this Euler equation with respect to a standard growth model is the appearance of obsolescence costs (negative impact of $g_{\widetilde{e}}$). These are due to the presence of embodied technological progress, which increases the user cost of capital because of the expected loss in its value derived from the fact that future technological improvements only affect new capital goods. An acceleration in the growth rate of embodied technological progress results, then, in a decrease in the overall growth rate.

Notice that g depends only on parameters remaining constant at the equilibrium path (AK technology). The growth rate is modified only when there are changes in the psychological discount factor, in the depreciation rate, in the aggregate marginal productivity ($z_0 e_0$) or a reassignment process in the stock of capital (variation in any of the parameters $\mu,\ \lambda,\ \gamma$). Since individual firms do not internalize the LBD externality, the individual marginal productivity of capital is a fraction $(1-\mu)$ of the aggregate marginal productivity ($z_0 e_0$).

The first order optimality conditions (34) and (35) are already written in intensive form. The envelope theorem conditions in stationary terms become:

$$\mathcal{W}^H_{N^c_t} \;=\; \frac{w^c_t}{C^h_t} \;-\; \frac{w^u_t}{C^h_t} \;+\; \beta \, \mathrm{E}_t \left[\, (1-\chi^c - p^c_t \; sc_t) \, \mathcal{W}^H_{N^c_{t+1}} - p^s_t \; sh_t \, \mathcal{W}^H_{N^{sh}_{t+1}} \, \right] \, , \tag{42}$$

$$\begin{aligned} \mathcal{W}^H_{N^{sh}_t} \;&=\; \frac{w^{sh}_t}{C^h_t} \;-\; \frac{w^u_t}{C^h_t} \;-\; \mathcal{D}\left(eo_t\right) \;+ \\ &+\;\; \beta \, \mathrm{E}_t \left[\, p^c_t \, (so_t - sc_t) \, \mathcal{W}^H_{N^c_{t+1}} + (1-\chi^s - so_t \; p^c_t - p^s_t \; sh_t) \, \mathcal{W}^H_{N^{sh}_{t+1}} \, \right] \, . \end{aligned} \tag{43}$$

The representative low-skilled household

Low-skilled unemployed workers must choose between searching for a simple job or doing some "domestic production". Their sole decision variable is the fraction of time el_t they devote to job search rather than to domestic activities since, by assumption, the low-skilled household accumulates no capital. Its optimization problem can thus be written as the following Bellmann equation:

$$\mathcal{W}^L_t = \max_{el_t} \left\{ (1-\alpha) \, \mathcal{U}\left(\frac{\widetilde{C^l_t}}{1-\alpha} \right) + \beta \, \mathrm{E}_t \left[\mathcal{W}^L_{t+1} \right] \right\} , \tag{44}$$

subject to (16-b) and the flow budget constraint:

$$\widetilde{C_t^l} = \widetilde{w_t^{sl}} N_t^{sl} + U_t^l \ \left[\widetilde{w_t^u} + (1 - el_t)\, \widetilde{y_t^d} \right] . \tag{45}$$

$\mathcal{W}_t^L$ is a function of the initial value of the state variable N_t^{sl}, $\widetilde{C_t^l}$ is the total amount consumed by the low-skilled household, $\widetilde{y_t^d}$ is the productivity of unemployed workers on domestic activities and it is proportional to aggregate marginal productivity.

The first order optimality condition can be written as follows:

$$\mathcal{U}_{\widetilde{C_t^l}}\ \widetilde{y_t^d} = \beta\, p_t^s\ sl_{el_t}\ \mathrm{E}_t \left[\mathcal{W}^L_{N^{sl}_{t+1}} \right] , \tag{46}$$

where $\mathcal{W}^L_{N^{sl}_{t+1}} = \frac{\partial \mathcal{W}^L_{t+1}}{\partial N_t^{sl}}$ is the instantaneous utility of an additional simple job at time $t+1$. From the envelope theorem we obtain:

$$\mathcal{W}^L_{N_t^{sl}} = \mathcal{U}_{\widetilde{C_t^l}}\ \left(\widetilde{w_t^{sl}} - \widetilde{w_t^u} - (1 - el_t)\, \widetilde{y_t^d} \right) + \beta \left(1 - \chi^s - p_t^s\ sl_t \right) \mathrm{E}_t \left[\mathcal{W}^L_{N^{sl}_{t+1}} \right] \tag{47}$$

We rewrite equations (45), (46) and (47) in terms of stationary variables:

$$C_t^l \ = \ w_t^{sl}\ N_t^{sl} \ + \ w_t^u\ U_t^l \ + \ \frac{\psi}{N_t}\ (1 - el_t)\ U_t^l\ , \tag{48}$$

$$\frac{1}{C_t^l} \ = \ \frac{N_t}{\psi}\ \beta\, \mathrm{E}_t \left[\ p_t^s\ sl_{el_t}\ \mathcal{W}^L_{N^{sl}_{t+1}} \ \right] , \tag{49}$$

$$\mathcal{W}^L_{N_t^{sl}} \ = \ \frac{w_t^{sl}}{C_t^l} \ - \ \frac{w_t^u}{C_t^l} \ - \ \frac{1}{C_t^l}\ \frac{N_t}{\psi} \ + \ \beta\, (1 - \chi^s - p_t^s\ sl_t)\ \mathrm{E}_t \left[\ \mathcal{W}^L_{N^{sl}_{t+1}} \ \right] . \tag{50}$$

Wage Determination

At the beginning of every period wages are renegotiated between intermediate firms and workers. Since there are three types of match (high-skilled worker on a complex or a simple job and low-skilled worker on a simple job) we will have three different levels of wages. All of them are determined through a Nash bargaining sharing surplus rule between the intermediate firm and its worker:

$$\mathcal{W}^H_{N_t^c} \ = \ \eta^c \left(\mathcal{W}^H_{N_t^c} + \mathcal{U}_{\widetilde{C_t^h}}\ \widetilde{W^F_{N_t^c}} \right) , \tag{51}$$

$$\mathcal{W}^H_{N_t^{sh}} \ = \ \eta^{sh} \left(\mathcal{W}^H_{N_t^{sh}} + \mathcal{U}_{\widetilde{C_t^h}}\ \widetilde{W^F_{N_t^{sh}}} \right) , \tag{52}$$

$$\mathcal{W}^L_{N_t^{sl}} \ = \ \eta^{sl} \left(\mathcal{W}^L_{N_t^{sl}} + \mathcal{U}_{\widetilde{C_t^l}}\ \widetilde{W^F_{N_t^{sl}}} \right) . \tag{53}$$

where η^i, for $i = c,\ sh,\ sl$ represent the workers bargaining powers, $\mathcal{W}^H_{N^i}\ \ \forall i = c, sh$, $\mathcal{W}^L_{N^{sl}}$ the utility surplus obtained, respectively, by high- and low-skilled households from having one additional member employed and $\widetilde{W^F_{N^i}}\ \ \forall i = c, sh, sl$ the marginal increase in the surplus obtained by the firm when filling a complex or a simple vacancy.

The Nash bargaining sharing rule can be rewritten as follows:

$$\mathcal{W}^H_{N^c_t} = \Lambda^c \, \frac{\widetilde{W^F_{N^c_t}}}{\widetilde{C^h_t}} \ , \qquad \mathcal{W}^H_{N^{sh}t} = \Lambda^{sh} \, \frac{\widetilde{W^F_{N^{sh}_t}}}{\widetilde{C^h_t}} \ , \qquad \mathcal{W}^L_{N^{sl}_t} = \Lambda^{sl} \, \frac{\widetilde{W^F_{N^{sl}_t}}}{\widetilde{C^l_t}} \ , \tag{54}$$

where $\Lambda^i = \frac{\eta^i}{1-\eta^i}\ \forall i = c,\ sh,\ sl$. Since in the balanced growth path both, $\widetilde{W^F_{N^i_t}}\ \forall i = c,\ sh,\ sl$ and $\widetilde{C^j_t}\ \forall j = h,\ l$, grow at rate g, their ratio is constant, *i.e.* $\mathcal{W}^H_{N^i_t}\ \forall i = c,\ sh$ and $\mathcal{W}^L_{N^{sl}_t}$ remain invariant in the equilibrium path.

4 Model Calibration and Simulations

In this section we calibrate the model and use deterministic simulation exercises to illustrate the properties of the model and gain insight on the effects of various types of shocks. The emphasis will be on the effects of labor force composition and embodied technological progress and their interactions with "institutional" settings over the period 1976-1996.

4.1 Specification and Calibration

The matching function on each labor market segment is represented by the usual Cobb-Douglas specification with constant returns to scale:

$$M^c_t = m^c \left(V^c_t\right)^{1-\lambda^c} \left(sc_t\, U^h_t + so_t\, N^{sh}_t\right)^{\lambda^c} \quad \text{and} \quad M^s_t = m^s \left(V^s_t\right)^{1-\lambda^s} \left(sh_t\, U^h_t + sl_t\, U^l_t\right)^{\lambda^s} \tag{55}$$

for complex and simple jobs respectively. As in many RBC models, we represent the instantaneous utility by the logarithm of per capita consumption expenditures. The leisure cost of on-the-job search is proportional to the amount of time spent and home productivity is assumed to be equal to a fraction ψ of aggregate productivity. More formally:

$$\mathcal{U}_t = \ln \widetilde{c}_t \ , \qquad \mathcal{D}_t = \tau\ eo_t \qquad \text{and} \qquad \widetilde{y^d_t} = \psi\, \frac{\widetilde{Y}_t}{N_t} \ . \tag{56}$$

Search efficiencies are represented by linear functions of the square root of the time devoted to search:

$$so_t = \phi_0^o + \phi_1^o \sqrt{eo_t} \qquad \text{On-the-job search efficiency} \tag{57}$$

$$sc_t = \phi_0^c + \phi_1^c \sqrt{eh_t} \quad \text{and} \quad sh_t = \phi_0^h + \phi_1^h \sqrt{1 - eh_t} \tag{58}$$

High-skilled unemployed search efficiencies

$$sl_t = \phi_0^l + \phi_1^l \sqrt{el_t} \qquad \text{Low-skilled unemployed search efficiency} \tag{59}$$

The parameters of the model are whenever possible set to values compatible with the available empirical evidence. The parameters for which no empirical estimates are available are chosen so as to reproduce the situation observed in Belgium in the mid nineties (1996). As in most EU countries, the Belgian economy was then neither in a recession nor in a boom. In terms of employment performance, the Belgian economy is in the EU average and quite representative of a typical "European" economy.

The numerical values of the parameters are reported in table 3. The reference period is the quarter. The elasticity of output with respect to capital coincides with the capital share of total income; it is set to the standard value $1 - \mu = 0.33$. The depreciation rate δ is set to 2.5%. We assume that high- and low-skilled workers are equally productive on simple jobs ($\nu = 1$). Calibrating the parameters describing the efficiency of the LBD processes turns out to be complicated due to the absence of empirical data concerning them. We take as reference the extended version of paper by Boucekkine, del Rio, and Licandro (2002) where they implement a quantitative exercise on the basis of the US economy. They estimate the ratio λ/μ to be around 0.75 before 1973 and around 0.90 after 1973. Even if in our case we are dealing with an European economy, we use these values as reference[11]. We set λ equal to 0.6 (then γ equals 0.06) so that in 1996 the relationship between λ and μ is around 0.9. The value of the constant $z_0 e_0$ is set so as to reproduce the average growth rate of the Belgian economy between 1976-1996 (period after the first oil shock). The effects of embodied technical progress on the relative demand for high- and low-skilled workers are determined by the values of parameters a_0 and a_1. These values are

[11] The important aspect for our simulations is not the absolute value of λ itself, but rather the variation it suffers between 1976 and 1996. Using the estimates provided in Boucekkine, del Rio, and Licandro (2002) as reference simply allows to assume a realistic value for the efficiencies of the technological progress learning processes.

chosen so as to reproduce the 1976 and 1996 values of the Cobb-Douglas coefficients $\mu\theta^c$ and $\mu\theta^s$ reported in Sneessens and Shadman (2000) for the Belgian economy.

	Symbol	Value	Symbol	Value
Production	$z_0 e_0$	0.19	μ	0.66
	λ	0.60	γ	0.06
	ν	1.00	δ	0.025
	a_0	251	a_1	0.87
Labor force composition	α	0.67		
Preferences	β	0.99	τ	0.35
Domestic productivity	ψ	0.19		
Search efficiencies	ϕ_0^l	0.10	ϕ_1^l	1.00
	ϕ_0^h	0.10	ϕ_1^h	1.00
	ϕ_0^c	0.20	ϕ_1^c	0.40
	ϕ_0^o	0.20	ϕ_1^o	0.40
Matching efficiencies	m^c	0.48	λ^c	0.50
	m^s	0.48	λ^s	0.50
Bargaining power	η^c	0.50		
	η^{sh}	0.50	η^{sl}	0.50
Vacancy costs	v_0^c	0.9	v_0^s	0.23
Job destruction rates	χ^c	0.03	χ^s	0.045
Average replacement ratio	bu	0.35		

Table 3: Numerical parameter values (year of reference: 1996)

We define the high-skilled group by an educational attainment level at least equal to a upper-secondary degree and set $\alpha = 0.67$, the 1996 value reported by Sneessens and Shadman (2000) for Belgium. As in most RBC models, consumer's psychological discount factor β is set to 0.99. The domestic productivity parameter ψ is fixed at 0.19 (*i.e.* the domestic productivity of a low-skilled worker equals 19% of the average aggregate productivity), a value which seems reasonable and gives a realistic relative wage ($w^{sl}/w^c = 64\%$, a value close to the relative wages estimated in the OECD (1996) for Belgium). We assume a Katz index[12] of 0.58 (as in Joseph, Pierrard, and Sneessens (2004)). This leads to a relative minimum wage of 0.4 which is below the 0.5 wage perceived by low-skilled workers in simple jobs. This result remains compatible with the Belgian evidence where only 2% of workers perceive the legal minimum wage.

[12]Defined as the ratio of the minimum wage to the average wage.

We have four search intensity equations, two for each segment of the labor market. We impose identical parameter values for a given segment, which leaves four values to fix. The simple job market search intensity coefficients have been chosen so as to have a sensitivity to labor market tightness of the order of magnitude estimated by Patacchini and Zenou (2003) (around 0.4). The complex market search intensity coefficients, the disutility parameter τ, the constants v_0^c and v_0^s and the two matching efficiency parameters are given values to reproduce the 1996 values of the high- and low-skilled unemployment rates (6.8% and 20.1%, respectively), the probabilities of filling a vacant complex or simple job (q_t^c and q_t^s, around 0.5; see Delmotte, Hootegem, and Dejonckheere (2001)) and the probabilities of finding a complex or a simple job (values p_t^s and p_t^c such that the probability to find a job is around 20% for a low-skilled worker and 40% for a high-skilled-worker; see Cockx and Dejemeppe (2004)).

We follow most authors and set the parameter determining the worker's share of a match surplus at the same value as the coefficient of unemployment in the matching function[13] (see for instance Merz (1995) and Andolfatto (1996)). The latter is set equal to 0.5, a value obtained in many empirical estimates of the Cobb-Douglas matching function (see Petrongolo and Pissarides (2001)). Calibrations for the average replacement ratio ($bu = 35\%$) and the two job destructions rates ($\chi^c = 3\%$ and $\chi^s = 4.5\%$) are based on estimations by Van der Linden and Dor (2001) for the Belgian economy. A result of this calibration exercise is that the proportion of high-skilled workers on simple jobs equals 6.3%.

4.2 Steady State Simulations

We examine[14] the steady state effects of four different types of shocks: one labor force composition change (in α), one productivity shock (an increase in the efficiency of the learning process in the investment good sector (λ)) and two types of wage shock (an increase in the unemployment benefits (w^u) and a subsidy to low-skilled wages (txs)). As it can be observed, the exogenous variables chosen to be shocked correspond, to policies (subsidies to low-skilled wages, increase in unemployment benefits,etc.) or stylized facts (development of embodied and disembodied tech-

[13]This choice is typically motivated by the so-called Hosios-Pissarides efficiency condition. It is worth noting that in our setup with job competition this condition may not be sufficient to ensure efficiency.

[14]We use the software Dynare developed at CEPREMAP (Paris) by Michel Juillard.

nological progress, increase in the proportion of high-skilled workers, etc.) observed over the last decades in many European countries. The results for the long run effects are summarized in tables 4-7. We briefly comment each exercise.

Labor force composition

A rise in α (the proportion of high-skilled workers in total labor force) increases the probability to fill a complex job in the equilibrium path, and thus stimulates the opening of complex vacancies. The higher amount of complex jobs deteriorates its marginal productivity, leading to a reduction in w^c. On the other side, the marginal productivity of simple jobs is stimulated (the relative wage improves), and the number of simple vacancies raises. The search effort of high- and low-skilled workers on the simple segment of the market ($1 - eh_t$ and el_t, respectively) increases. This results in an upturn of the ladder effect. All in all both unemployment rates decrease.
The equilibrium growth rates are not affected by a change in α since none of the parameters being determinants of g is modified.

	Low-skilled search efficiency	High-skilled unemployment rate	Low-skilled unemployment rate	Aggregate unemployment rate	Relative wage	Ladder effect	Growth rate
Benchmark simulation corresponding to the year 1996	39.2%	7.0%	20.5%	11.5%	63.7%	6.3%	1.11%
Proportion of high-skilled workers: $\alpha = 0.670 \rightarrow \alpha = 0.700$	+16.2	-0.4	-3.4	-1.7	+3.5	+3.2	+0.00

Table 4: Steady state effects (deviations from the benchmark) of an increase in the proportion of high-skilled workers.

Biased technological shock

We now consider the effects of an improvement in the learning efficiency of embodied technological progress. More particularly, we introduce an increase in λ of 1.5%. From equation (41) we observe that the acceleration in the growth rate of embodied technological progress (coming from the upturn in λ) negatively affects the equilibrium growth rate g[15] (productivity slowdown). At the same time, because of the new technologies-skill complementarity relationship (equation 5), the increase in λ raises the share of complex jobs in the production function leading the firm to open more complex vacancies. The higher θ_t^c avoids the appearance of decreasing returns on complex jobs and w^c increases. Search efforts on the complex segment are stimulated.

	Low-skilled search efficiency	High-skilled unemployment rate	Low-skilled unemployment rate	Aggregate unemployment rate	Relative wage	Ladder effect	Growth rate
Benchmark simulation corresponding to the year 1996	39.2%	7.0%	20.5%	11.5%	63.7%	6.3%	1.11%
Efficiency of the embodied technical progress learning process: $\lambda = 0.600 \rightarrow \lambda = 0.610$	-9.0	+0.2	+3.5	+1.0	-1.2	-0.7	-0.03

Table 5: Steady state effects (deviations from the benchmark) of a biased technological progress.

On the simple side, the evolution is the opposite. The reduction in θ_t^s has a negative impact on the productivity of simple jobs. Firms open less simple vacancies and the number of simple jobs falls. Wages are reduced and the search effort on the simple segment too. Low-skilled unemployment increases and so does high-skilled unemployment since the demand for high-skilled in complex jobs is unable to compensate the downturn in the ladder effect.

Replacement ratio

An increase in the replacement ratio mainly affects low-skilled workers, whose consumption is constrained by their revenue. A higher replacement ratio, raises their reservation wage. This pushes wages of low-skilled workers up and decreases the supply of simple vacancies. The downturn in simple jobs raises their marginal productivity which increases their relative wages even more. This stimulates search efforts on the simple segment (rise in el_t and the ladder

[15] The more important embodied technological progress, the higher will be the user cost of capital due to the expected loss in its value coming from the fact that future technological improvements will only affect new capital stock.

effect).

	Low-skilled search efficiency	High-skilled unemployment rate	Low-skilled unemployment rate	Aggregate unemployment rate	Relative wage	Ladder effect	Growth rate
Benchmark simulation corresponding to the year 1996	39.2%	7.0%	20.5%	11.5%	63.7%	6.3%	1.11%
Unemployment benefit: $w^u = 0.246 \rightarrow w^u = 0.271$	-10.6	+0.2	+3.0	+1.1	+2.2	+0.3	+0.00

Table 6: Steady state effects (deviations from the benchmark) of an increase in unemployment benefits.

The marginal productivity of complex jobs is negatively affected by the reduction in the number of simple jobs, which leads firms to open less complex vacancies. Both unemployment rates increase. The equilibrium growth rate of the economy remains constant since none of its determinants has been modified.

Subsidy to low-skilled wages

A target policy giving to firms a proportional subsidy to low-skilled wages (and financed from high-skilled wages) turns out to be very beneficial for low-skilled workers (see Pierrard (2004) for another quantitative example on the Belgian economy) in terms of unemployment reductions, and somewhat damaging for high-skilled workers. Indeed, the subsidy increases the marginal value the firm obtains from low-skilled workers, which stimulates the opening of simple vacancies as well as the rise of low-skilled wages. Search efforts of low-skilled workers on the simple segment are thus increased. In contrast, the reduction in the relative wage earned by the high-skilled on the simple segment (the marginal surplus the firm obtains from hiring a high-skilled worker in a simple job is now relatively smaller) decreases their search effort in this labor market segment (ladder effect falls).

Even if the increase in the number of simple jobs improves the marginal productivity of complex jobs, the marginal value the firm obtains from a complex job is now smaller in relative terms. There are, thus, less complex vacancies opened and complex employment is reduced. This, to-

	Low-skilled search efficiency	High-skilled unemployment rate	Low-skilled unemployment rate	Aggregate unemployment rate	Relative wage	Ladder effect	Growth rate
Benchmark simulation corresponding to the year 1996	39.2%	7.0%	20.5%	11.5%	63.7%	6.3%	1.11%
Proportional subsidy to low-skilled wages: $txs = 0.000 \rightarrow txs = 0.100$	+28.0	+0.2	-5.5	-1.7	+4.3	-0.6	+0.00

Table 7: Steady state effects (deviations from the benchmark) of a proportional subsidy to low-skilled wages.

gether with the downturn in the ladder effect results in an increase of high-skilled unemployment rates. Growth remains unaffected.

4.3 Historical Comparison

In the first row of table 8 we reproduce, the 1996 values of the proportion of high-skilled workers in the labor force (α), the share of complex jobs in the production function ($\mu\theta^c$), the net skill-bias[16], the high- and low-skilled unemployment rates, the relative wage (w^{sl}/w^c) and the ladder effect. The value of the average growth rate g corresponds to the period going from 1976 to 1996. On the second row we display the change observed for all variables over the period 1976-1996. The variation of g corresponds to the comparison of the average growth rates corresponding to the periods 1950-1976 and 1976-1996, since what we analyze is the fall in average growth rates before and after the oil shock (annual growth rates decreased by 2.4 percentage points).

The last two rows of table 8 contain the values predicted by our model for 1996 as well as the predicted variation between 1976 and 1996 when we introduce the observed rise in the proportion of high-skilled workers (it evolved from 21.5% of the total labor force in 1976 to 67% in 1996) and the increase in the importance of embodied technological progress. The variation in λ is finally chosen to reproduce the Belgian productivity slowdown (there is no empirical evidence on this parameter for the Belgian economy). Moreover, the selected value of λ (λ/μ=0.61)

[16] It is defined as the ratio of the relative productivity coefficient $\mu\ \theta_t^c/(\mu\ \theta_t^s)$ and the relative labor force $\alpha/(1-\alpha)$.

	Prop. of high-skilled in the labor force	Share complex jobs in the production function	Net Skill Bias	High-skilled unemployment rate	Low-skilled unemployment rate	Relative wage	Ladder effect	Average growth rate	
Actual data									
1996	0.67	0.51	1.59	6.8%	20.1%	66%	n.a.	1976-96	1.1%
1976-96 absolute deviations	+0.45	+0.33	+0.20	+2.1	+13.3	-0.0	n.a.	1950-76 to 1976-96	-0.6
Model's simulation									
1996	0.67	0.52	1.75	7.0%	20.5%	63.7%	6.3%		1.1
1976-96 absolute deviations	+0.46	+0.32	+0.20	+1.7	+13.3	-0.6	+4.9		-0.6

Table 8: Skill-bias, unemployment rates, relative wages, ladder effect and productivity slowdown in Belgium: comparing actual and simulated data.

for 1976 manages to be quite coherent with the simulations implemented by Boucekkine, del Rio, and Licandro (2002) for the U.S. economy. The model performs well in reproducing not only the productivity slowdown (notice that the reference period is the quarter, therefore a reduction in g of around 0.6 points per quarter implies that annual growth rates decreased around 2.5 points which is coherent with the estimations given for Germany or France in table 2 or for Belgium here above), but also the increase in unemployment rates and the relative wage stability. Incorporating endogenous growth to the model developed in Moreno-Galbis and Sneessens (2004) keeps then its good properties in terms of a realistic representation of the labor market and, at the same time, it permits to take into account growth issues (productivity slowdown).

Given the results in the previous sections, it can be concluded that the increase in the efficiency of the embodied technological progress is the main factor responsible of the rise in unemployment rates. Actually, if the proportion of high-skilled in the labor force (α) had increased from 21.5% to 67%, everything else remaining constant, we would have observed a decrease in both unemployment rates and a huge upturn in the ladder effect. On the other hand, *ceteris paribus*, a rise in the efficiency of embodied technical progress (λ), results in higher unemployment rates because of the decreased demand for simple jobs which also leads to a reduction in the ladder effect (this affects high-skilled workers). The combination of both shocks (upturn in α and λ)

increases high- and low-skilled unemployment rates as well as the ladder effect, this last one raising by much less than in the case where only α was modified.

In terms of the European stylized facts, the previous results can be interpreted as follows. The post-1975 period has been characterized by a higher efficiency in the investment good sector (upturn in λ) and a consequent reduction in the learning efficiency of the final good sector (downturn in γ). This technological reassignment favoring embodied technological progress (upturn in λ and downturn in γ) and inducing, thus, a rise in the obsolescence costs, has increased the user cost of capital and lowered the growth rate permanently. On the other hand, because of the new technologies-skill complementarity relationship, the acceleration of embodied technical progress has stimulated complex jobs creation and simple jobs destruction. The increased demand for high-skilled workers in complex jobs has been more than satisfied by the large upturn in their supply. In contrast, the fall in the importance of the low-skilled labor force has not been enough to compensate the massive destruction of simple jobs and the increased job competition. Low-skilled unemployment rates have raised.

4.4 Alternative Policy Scenarios

During the beginning of the nineties crisis, many European governments started questioning the suitability of the welfare state in the economic and social context corresponding to the end of the twentieth century. Particularly, unions have been forced to negotiate with governments measures tending to reduce the unemployment benefits or restricting the access to them. We analyze now two different historic scenarios that could have arisen if, as a consequence of the beginning of the nineties crisis, welfare policies regarding unemployment had been modified.

Tables 9 and 10 report the predicted results by the benchmark simulation for 1996 and the predicted results after the policy measure is implemented during a period of time (final state). The tables display the simulated values for the proportion of high-skilled workers in the labor force, the share of complex jobs in the production function, unemployment rates, relative wages, the ladder effect, the welfare levels and the average growth rates. We use as welfare indicators the utility functions of each type of worker: $\mathcal{U}^h = ln(C_t^h/\alpha) - N_t^{sh}\, \mathcal{D}\left(eo_t\right)/\alpha$ proxies the high-skilled welfare level and $\mathcal{U}^l = ln(C_t^l/(1-\alpha))$ that of low-skilled.

	Prop. of high-skilled in the labor force	Share complex jobs in the production function	High-skilled unemploy. rate	Low-skilled unemploy. rate	Relative wage	Ladder effect	High-sk. welfare level	Low-sk. welfare level	Average growth rate
Model's simulation for 1996: Benchmark simulation									
1996	0.67	0.52	7.0%	20.5%	63.7%	6.3%	-0.30	-0.70	1.1
Gradual decrease over 10 years in the replacement ratio until bu=25%									
Final state	0.67	0.52	6.2%	12.5%	58.0%	6.4%	-0.24	-0.82	1.1

Table 9: Comparison of the benchmark simulation for Belgium 1996 with a policy scenario where the replacement ratio is reduced to 25%.

We start considering a situation where the replacement ratio is progressively reduced from 35% to 25% over forty periods (ten years). More precisely, the calibrations we have adopted for the initial α correspond to the beginning of the nineties crisis ($\alpha = 0.60$), and the variation in λ is chosen so as to reproduce the slight decline (of around 0.55 percentage points) in the average growth rates observed for Belgium between the early and the late nineties. The dotted lines in figure 2 represent the evolution of different economic variables over the nineties if, together with the progressive increase in the proportion of high-skilled workers (α) and in the efficiency of embodied technological progress (λ), the government had gradually reduced the replacement ratio by ten percentage points, down to 25%. The solid lines in figure 2 correspond to the benchmark simulation, *i.e.* the evolution that would have been observed if welfare policies had remained unchanged (bu =35%) during the forty periods.

The downturn in the unemployment benefit mainly affects low-skilled workers whose consumption is constrained by their revenue at each moment of time (they do not save). The fall in their outside option leads low-skilled workers to reduce their reservation wage which permits firms to offer them lower wages. There are more simple vacancies opened and the demand for low-skilled workers is then stimulated. This results in lower unemployment rates with respect to the benchmark simulation during the transition period.

High-skilled unemployment rates remain, all through the transition period , at a lower level than in the benchmark simulation. This is probably explained by the higher ladder effect observed

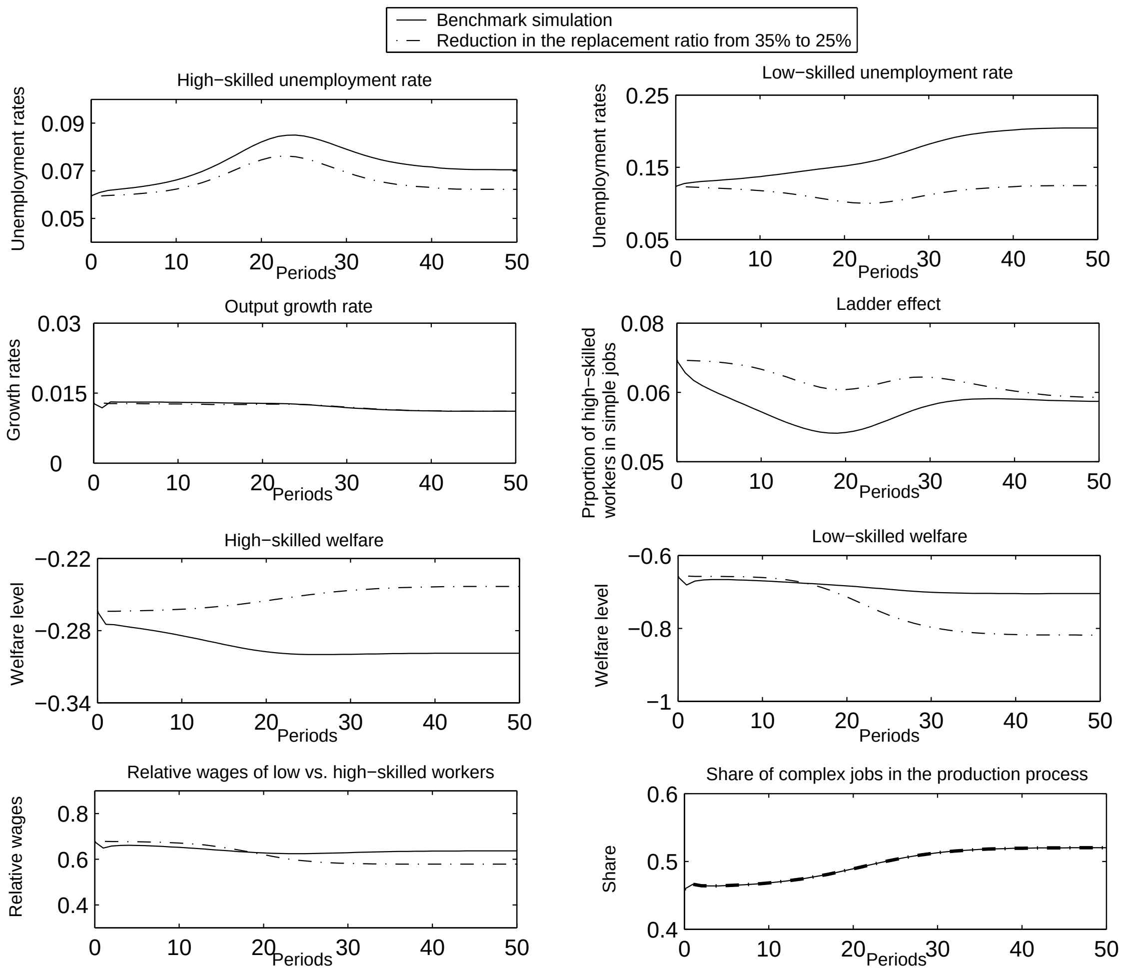

Figure 2: Labor flows, output growth rates, welfare level and relative wages: comparing the benchmark transition with the transition observed if the replacement ratio is progressively reduced from 35% to 25%.

under the implemented welfare policy, *i.e.* the opening of more simple jobs leads high-skilled workers to look for a job in the simple segment even if the wage they earn is lower. The initial upward trend observed for high-skilled unemployment rates in the two represented simulations is due to the fact that the economy is not able to absorb immediately the rise in the number of high-skilled workers coming from the upturn in α. From around the 20th period (5 years after the beginning of the policy), the increased demand for high-skilled labor derived from the gains in the relative marginal productivity of complex jobs (new technologies-skill complementarity relationship) manages to compensate their larger supply and high-skilled unemployment rates start decreasing.

	Prop. of high-skilled in the labor force	Share complex jobs in the production function	High-skilled unemploy. rate	Low-skilled unemploy. rate	Relative wage	Ladder effect	High-sk. welfare level	Low-sk. welfare level	Average growth rate
Model's simulation for 1996: Benchmark simulation									
1996	0.67	0.52	7.0%	20.5%	63.7%	6.3%	-0.30	-0.70	1.1
Combining a gradual decrease over 10 years in the replacement ratio until bu=25% and a gradual increase in the proportional subsidy to low-skilled wages up to 10%									
Final state	0.67	0.52	6.1%	9.7%	63.1%	6.5%	-0.26	-0.76	1.1

Table 10: Comparison of the benchmark simulation for Belgium 1996 with a policy scenario where the replacement ratio is reduced to 25% and a proportional subsidy of 10% of low-skilled wages is given to firms.

Because wages in simple jobs are deteriorated and those in complex jobs improved, relative wages fall. This leads to an increase in high-skilled welfare levels (they can consume more) and to a deterioration in low-skilled welfare levels. Even if at the end of the 10 years measure, unemployment rates of both workers categories are lower than if the replacement ratio had been maintained at 35% (see table 9), inequality is much higher in terms of wages and welfare.

Such a policy measure is thus not likely to be implemented by any European government wishing to be re-elected unless this policy is associated with another measure trying to compensate low-skilled workers for this loss in welfare. Here we assume that, in exchange for the progressive reduction of ten percentage points in the replacement ratio, a proportional subsidy of 10% of low-skilled wages (given to the firms) is also gradually introduced at the same pace, this measure being financed on the basis of high-skilled wages, *i.e.* : at the beginning of the nineties the government decides that, over the ten coming years, the replacement ratio will be progressively reduced from 35% to 25% and a proportional subsidy to low-skilled wages will be increased, at the same pace, from 0% to 10%.

In figure 3 the solid lines correspond again to the benchmark simulation, where only α and λ are rising to attend their end-of-the-nineties value, while the dotted lines represent the transition under the implemented policies (together with the rise in α and λ, the replacement ratio is reduced and the subsidy to low-skilled wages increased). Comparing figures 2 and 3 we notice that the introduction of the subsidy mainly favors low-skilled workers, whose demand is stimulated and their relative wages improved.

While the transition of high-skilled unemployment rates remains almost unaffected with respect to figure 2, low-skilled unemployment rates follow now a decreasing trend. Moreover, the introduction of a subsidy almost dissipates the inequalities in wages and in relative consumption levels (the relative consumption of low-skilled workers falls by 0.03 points with respect to the benchmark simulation) that were generated by the downturn in the replacement ratio. To completely eliminate these inequalities it would be necessary to increase the subsidy by a bigger amount than the decrease of the replacement ratio (here the subsidy is increased by ten percentage points and the replacement ratio decreased by ten percentage points[17]).

Comparing tables 9 and 10 we conclude that an appropriate combination of two policies consisting in reducing the replacement ratio while increasing the subsidies on low-skilled wages can lead to a Pareto improving situation, where unemployment rates are lower and inequalities are not increased.

[17] Convergency problems with the numerical simulations prevent us from implementing the case where the increase in the subsidy to low-skilled wages is bigger than the decrease of the replacement ratio.

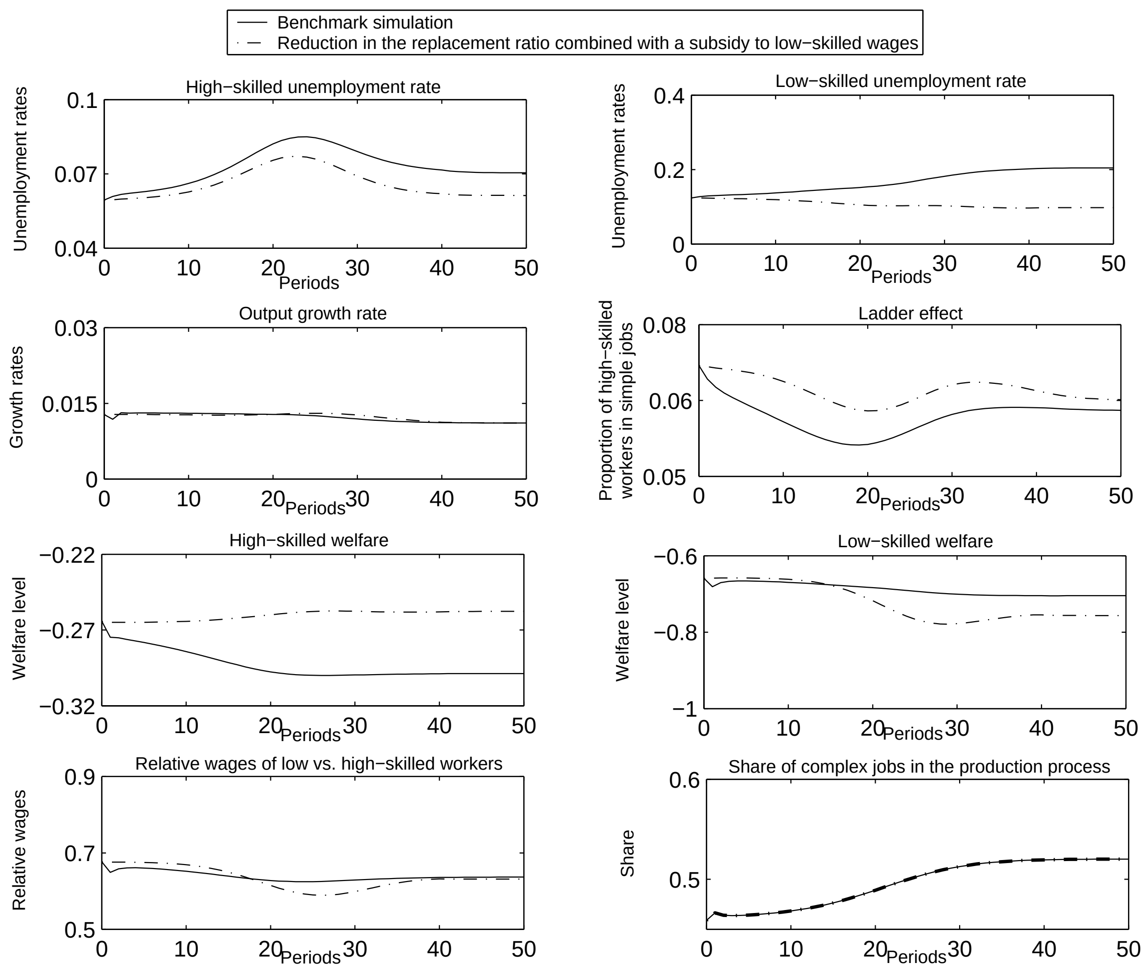

Figure 3: Labor flows, output growth rates, welfare level and relative wages: comparing the benchmark transition with the transition observed if the replacement ratio is progressively reduced from 35% to 25% and a proportional subsidy of 10% to low-skilled wages is also gradually introduced.

5 Conclusions

Over the last decades, particularly after the first oil shock, most European economies have been characterized by the increase in unemployment rates, the stability in relative wages and the decrease in per capita output growth rates (productivity slowdown). Even if these stylized facts have arisen together most of the existing literature has treated them separately. That is, models of economic growth have generally assumed full employment and labor market models have not generally considered growth issues.

Since the beginning of the nineties there is, however, an emerging literature trying to explain the interactions between unemployment and growth, most times in a comparative static analysis. Our paper constitutes the first attempt to deal simultaneously with the observed stylized facts about unemployment, wages and growth in European economies. Our main contribution consists, therefore, in taking into account both, European labor market and growth issues in a dynamic setup. We build an intertemporal general equilibrium model with endogenous growth and search frictions in the labor market. We calibrate it on the basis of Belgian data and verify its ability to reproduce not only the labor market behaviors in this country between 1976 and 1996 (increase in high- and low-skilled unemployment rates by 2 and 13.3 percentage points, respectively, and the relative wage rigidity) but also the evolution in growth rates (average per capita output growth rate has decreased by 2.5 percentage points per annum). We finally analyze from a dynamic perspective the implications of different policy measures.

Evidently the model presents some limitations. Particularly, the specification of new technologies-skill complementary relationship seems somewhat *ad hoc*. On the other side, this paper does not consider minimum wages while they have been a key determinant in European unemployment evolution. Incorporating them will certainly enrich the results (even if our model already incorporates a source of wage rigidity through the domestic production).

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Appendix: Equilibrium Growth Rates

This appendix explains the procedure followed to compute the equilibrium growth rates g and $g_{\widetilde{K}}$. The uniqueness of the growth path is guaranteed by the adopted calibration which implies the positivity of K_0, $\widetilde{C_t^h}$ and g and, thus, a bounded utility. We start computing the equilibrium growth rates associated to the final good production sector and capital sector (law of motion of capital). We then continue with the intermediate firms. The households optimality conditions permit to determine the Euler equation. We finish with the wage bargaining conditions.

The final firm

The production process as well as the marginal productivity of the different types of jobs can be easily written in terms of growth rates[18]:

$$
\begin{aligned}
g_{\widetilde{Y}_t} &= (1-\lambda)\, g_{\widetilde{K_t}} + \lambda\, g_{N_t^*} \ , &(60)\\
g_{N_t^*} &= \theta_t^c\, g_{N_t^c} + \theta_t^s\, g_{N_t^{s*}} \ , &(61)\\
g_{\widetilde{F_{N_t^c}}} &= g_{\widetilde{Y}_t} - g_{N_t^c} \ , &(62)\\
g_{\widetilde{F_{N_t^{sh}}}} &= g_{\widetilde{Y}_t} - g_{N_t^{s*}} \ , &(63)\\
g_{\widetilde{F_{N_t^{sl}}}} &= g_{\widetilde{Y}_t} - g_{N_t^{s*}} \ , &(64)
\end{aligned}
$$

where $N_t^* = (N_t^c)^{\theta_t^c}\ (N_t^{s*})^{\theta_t^c}$ and $N_t^{s*} = N_t^{sh} + \nu\ N_t^{sl}$. Because in the equilibrium path $g_{N_t^*}$, $g_{N_t^c}$, $g_{N_t^s}$ and $g_{N_t^{s*}}$ equal zero, we have a balanced path characterized by: $g_{\widetilde{Y}} = g_{\widetilde{F_{N^c}}} = g_{\widetilde{F_{N^{sh}}}} = g_{\widetilde{F_{N^{sl}}}} = g$ and $g_{\widetilde{K}} = \frac{g}{1-\lambda}$.

Equations (4) written in growth terms become:

$$
g_{\widetilde{e}_t} = \lambda\, g_{\widetilde{K_t}} - \lambda\, g_{N_t^*} \qquad \text{and} \qquad g_{\widetilde{z}_t} = \gamma\, g_{\widetilde{K_t}} - \gamma\, g_{N_t^*} \tag{65}
$$

so that in the equilibrium we find: $g_{\widetilde{e}} = \lambda\ \frac{g}{1-\lambda}$ and $g_{\widetilde{z}} = \gamma\ \frac{g}{1-\lambda}$.

The law of motion of capital is redefined as:

$$
g_{\widetilde{K_t}} = z_0 e_0\ \frac{\widetilde{I}_t}{\widetilde{Y}_t} - \delta\ . \tag{66}
$$

Since at the equilibrium path $g_{\widetilde{K_t}}$ is constant, $\frac{\widetilde{I}_t}{\widetilde{Y}_t}$ must be constant, that is: $g_{\widetilde{I}} = g_{\widetilde{Y}} = g$.

[18] Notice that we are working in discrete time, therefore to put the expression in growth terms, we compute logarithms and we subtract the first difference.

The intermediate firms

We determine now the growth rates of each variable involved in the intermediate firms' equilibrium conditions, once the free entry condition is applied ($W^V_{N^c_t} = W^V_{N^s_t} = 0$). We look first to the complex side of the labor market. By simply writing equation (19) in growth terms it is found that, at the equilibrium, $g_{\widetilde{b^c}} = g_{\widetilde{W^F_{Nc}}}$. Furthermore, replacing (19) and (30) in equation (20), leads, after simple algebra, to the following equilibrium result: $g_{\widetilde{w^c}} = g_{\widetilde{F_{Nc}}} = g$. Because vacancy costs associated to complex jobs are indexed to $\widetilde{w^c}$, we also have that $g_{\widetilde{b^c}} = g_{\widetilde{v^c}} = g_{\widetilde{W^F_{Nc}}} = g$.

In the simple side we follow a similar procedure and replace conditions in (30) in equations (22) and (23). After simple algebra we deduce that, at the balanced growth path, $g_{\widetilde{w^{sh}}} = g_{\widetilde{W^F_{Nsh}}} = g_{\widetilde{F_{Nsh}}} = g_{\widetilde{w^{sl}}} = g_{\widetilde{W^F_{Nsl}}} = g_{\widetilde{F_{Nsl}}} = g$. Since vacancy costs associated to simple jobs are indexed to the average simple wage we have that $g_{\widetilde{b^s}} = g_{\widetilde{v^s}} = g$.

The households

The budget constraint of the economy can be represented as follows:

$$\widetilde{Y_t} - \widetilde{v_t} = \widetilde{C_t} + \widetilde{I_t} + \widetilde{T_t} \tag{67}$$

where $\widetilde{v_t}$ represents the total vacancy costs (the costs of opening complex vacancies ($\widetilde{v^c_t}$) plus the costs of opening a simple vacancies ($\widetilde{v^s_t}$)), $\widetilde{C_t}$ the total consumption (that of high-skilled workers plus that of low-skilled workers) and $\widetilde{T_t}$ represents a lump-sum tax levied on the high-skilled household to finance government expenditures.

Because taxes and vacancy costs are indexed to wages, at the equilibrium they grow at rate g. Using the law of motion of capital, we have also shown that, at the balanced path, investment and output must grow at rate g. Finally, given these results and the fact that we are using a Cobb-Douglas production function and a logarithmic utility function we have that, at the equilibrium growth path, $g_{\widetilde{C_t}} = g$.

The budget constraint can now be redefined in terms of stationary variables:

$$1 = \frac{\widetilde{v^c_t}}{\widetilde{Y_t}} + \frac{\widetilde{v^s_t}}{\widetilde{Y_t}} + \frac{\widetilde{C^h_t}}{\widetilde{Y_t}} + \frac{\widetilde{C^l_t}}{\widetilde{Y_t}} + \frac{\widetilde{I_t}}{\widetilde{Y_t}} + \frac{\widetilde{T_t}}{\widetilde{Y_t}}\ . \tag{68}$$

From the optimality condition $\mathcal{U}_{C_t^h} = \beta \; \mathrm{E}_t \left[\left(1 + r_{t+1}\right) \mathcal{U}_{C_{t+1}^h} \right]$ we derive both, the value of the discount factor at the equilibrium path and the Euler equation. Regarding the former, notice first that, in a growth context, the firm's discount factor is not simply the interest rate, but the "effectiveness rate", which is composed by two terms:

$$1 + r_{t+1} = \widetilde{e}_t \; \widetilde{c_{t+1}^K} + (1 - \delta) \; \frac{\widetilde{e}_t}{\widetilde{e_{t+1}}} \tag{69}$$

The first term on the right hand side represents the rents obtained by the firm in period $t+1$ if it had invested one unit in period t (for each invested unit, the firm gets e_t units of capital and each one produces $\widetilde{c_{t+1}^K}$ rents in period $t+1$). The second term on the right hand side represents the increase in the value of capital obtained by the firm when investing one unit in a growth context. In words, for every unit of good invested in period t, the firm obtains $(1 - \delta) \; \widetilde{e}_t$ units of capital in period $t + 1$. Furthermore, these units are valued at price $\frac{1}{\widetilde{e_{t+1}}}$.

The value of the discount factor at the balanced growth path equals $1+r = \frac{1}{\beta}(1+g_{C_t^h}) = \frac{1}{\beta}(1+g)$. On the other hand, from the household's optimality condition, and knowing that $\beta = \frac{1}{1+\rho}$, where ρ can be interpreted as a rate of time preference, we also derive the Euler equation providing the equilibrium growth rate value:

$$\widetilde{C_{t+1}^h} = \frac{1}{1+\rho} \; \widetilde{C_t^h} \left(\frac{\widetilde{e}_t}{\widetilde{e_{t+1}}} + (\widetilde{e_{t+1}} \; \widetilde{c_{t+1}^K} - \delta) \; \frac{\widetilde{e}_t}{\widetilde{e_{t+1}}} \right), \tag{70}$$

$$g_{\widetilde{C_{t+1}^h}} = \frac{1}{1+\rho} \left(\frac{\widetilde{e}_t}{\widetilde{e_{t+1}}} - 1 - \rho + (\widetilde{e_{t+1}} \; \widetilde{c_{t+1}^K} - \delta) \; \frac{\widetilde{e}_t}{\widetilde{e_{t+1}}} \right), \tag{71}$$

$$g_{\widetilde{C_{t+1}^h}} = \frac{1}{1+\rho} \left(- \frac{\frac{\widetilde{e_{t+1}} - \widetilde{e}_t}{\widetilde{e}_t}}{\widetilde{e_{t+1}}} \; \widetilde{e}_t - \rho + (\widetilde{e_{t+1}} \; \widetilde{F_{K_{t+1}}} - \delta) \; \frac{1}{1 + g_{\widetilde{e_{t+1}}}} \right). \tag{72}$$

Replacing $\widetilde{F_{K_{t+1}}}$, $\widetilde{e}_t$ and β by their expressions we obtain:

$$g_{\widetilde{C_{t+1}^h}} = \frac{\beta}{(1 + g_{\widetilde{e_{t+1}}})} \left[\; z_0 \; e_0 \; (1 - \mu) \; - \; \frac{1 - \beta}{\beta} \; (1 + g_{\widetilde{e_{t+1}}}) \; - \; \delta \; - \; g_{\widetilde{e_{t+1}}} \; \right], \tag{73}$$

which at the equilibrium path becomes:

$$g = \frac{\beta}{(1 + g_{\widetilde{e}})} \left[\; z_0 \; e_0 \; (1 - \mu) \; - \; \frac{1 - \beta}{\beta} \; (1 + g_{\widetilde{e}}) \; - \; \delta \; - \; g_{\widetilde{e}} \; \right] \tag{74}$$

Wage determination

Writing equations (51), (52) and (53) in growth terms leads to:

$$
\begin{aligned}
g_{\mathcal{W}^H_{N^c_t}} &= g_{\widetilde{W^F_{N^c_t}}} - g_{\widetilde{C^h_t}} \,, &(75)\\
g_{\mathcal{W}^H_{N^{sh}_t}} &= g_{\widetilde{W^F_{N^{sh}_t}}} - g_{\widetilde{C^h_t}} \,, &(76)\\
g_{\mathcal{W}^H_{N^{sl}_t}} &= g_{\widetilde{W^F_{N^{sl}_t}}} - g_{\widetilde{C^l_t}} \,. &(77)
\end{aligned}
$$

Therefore, at the equilibrium $g_{\mathcal{W}^H_{N^c}} = g_{\mathcal{W}^H_{N^{sh}}} = g_{\mathcal{W}^L_{N^{sl}}} = 0$. The intuition behind this result is that wages increase at the same rate as the marginal utility decreases leaving the actualized value of the instantaneous utility of an additional job constant at the equilibrium path.